GS St Philippe Neri Term 2 2025-2026

SUBJECT: MATHEMATICS

Holiday Package Class: S6 MPG, MCB & PCM

SECTION A Subjective type questions

1) **E**arthquake magnitude M and intensity I are related by $log\, I = 1.5M + 2$.

(a) Evaluate intensity for M=6. **(b)** Determine magnitude if intensity I=1000.

(c) Assess whether a magnitude 7 earthquake is twice as intense as magnitude 6.

2) **(a)** Construct an exponential growth model for a bacterial culture that doubles every 4 hours.

(b) Create a logarithmic decay model for a radioactive isotope with half-life 10 days.

(c) Design an exponential model for savings in a bank with continuous interest rate 5% per year. **(d)** Suppose that the amount of money in a bank is given by $P(t) = 500 + 100sin(t)$ where t is in years. During the first 5 years in which the account is opened when is the amount of money in the account decreasing?

3) **Creating Trigonometric , SHM & Optical Scenarios**

(a) Create a trigonometric equation whose solution is $x = 90^{\circ}$.
(b) Construct a simple inequality involving $sin\, x$ valid in the first quadrant.
(c) Construct an equation of displacement for SHM with amplitude 5 cm.
(d) Using Snell's law, assess whether refraction occurs when light is incident normally on glass.
(e) Create and solve a trigonometric inequality whose solution lies strictly in the second quadrant.

4) **(a)** State the general solution of $sin\, x = sin\, \alpha$. **(b)** Find the domain of $\sqrt{sin\, x}$ **(c)** Find the $10th$ term of the sequence defined by $a_1 = 3$ and $a_{n+1} = 2a_n - 1$.

(d) Find the sum of the first 12 terms of the arithmetic sequence whose 3rd and 7th terms are 10 and 22 respectively. **(e)** Explain the effect of squaring both sides of a trigonometric equation

(f) If the zeros of $f(x)$ are 3 and -4, What are the zeros of the function $g(x) = f\left(\frac{x}{3}\right)$?

5) a) Write the formula for linear interpolation

b) Describe the principle of locating a root using sign change.

c) Check whether displacement and acceleration are in the same direction in SHM

d) A student claims the solution of $\tan 2x = \tan x$ is $x = k\pi$. Evaluate the claim

e) Evaluate $\tan(\arctan 3 + \arctan 2)$ without using a calculator

f) Find the probability of obtaining at least 2heads when a coin is tossed thrice.

6) Given $f(x) = x^3 - x - 1$.

(a) Show that a root lies between $x = 1$ and $x = 2$.

(b) Find the first approximation using bisection method.

(c) Find the second approximation.

(d) Write the Newton–Raphson iteration formula.

(e) Take initial guess $x_0 = 1.5$ and find x_1 using Newton–Raphson iteration.

7) i) Solve the equation $f^{-1}(x) = gf(x)$ where f and g are defined by

$$f: x \longmapsto 3x + 2; \quad g: x \longmapsto 4x - 12$$

ii) Perform 3 iterations It is given that a is a positive constant.

(a) (i) Sketch on a single diagram the graphs of $y = |2x - 3a|$ and $y = |2x + 4a|$

(ii) State the coordinates of each of the points where each graph meets an axis.

(b) Solve the inequality $|2x - 3a| < |2x + 4a|$

of the bisection method to estimate the value of $\sqrt[3]{7}$ starting with $x_0 = 1.5$

iii) If $\sin 36^\circ \cos 12^\circ = p$ and $\cos 36^\circ \sin 12^\circ = q$, determine an expression in terms of p and q for $a)$ $\sin 48^\circ$ $\quad$ $b)$ $\cos 48^\circ$

iv) Three numbers, A ,B ,C in that order, are in geometric progression with common ratio *r* . Given further that *A*, 2*B*, *C* in that order are in arithmetic progression, determine the possible values of *r*.

8) a)Solve $AX - 2M = 0$ if A=$\begin{pmatrix} 1 & 2 & 0 \\ 1 & -1 & 2 \\ 2 & 0 & 1 \end{pmatrix}$ and M=$\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$

b) diagonalise A

c)find the inverse of A by Cayley Hamilton

d) find the inverse of A by Gaussian elimination method

9) The following table gives a number of advertisement (x_i) and the sales in hundreds of dollars (y_i) of a certain sport company.

(x_i)	1	2	3	4	5	6
(y_i)	41	50	54	54	57	63

a) Find the standard deviation for (x_i) and (y_i)
b) Calculate the correlation coefficient r
c) Find the equation of regression line Y on X
d) For 7 number of advertisements. Estimate the volume of sales

10) The population of a particular country consists of three ethnic groups. Each individual belongs to one of the four major blood groups. The accompanying joint probability table gives the proportions of individuals in the various ethnic group/blood group combinations

	Blood group			
Etic group	O	A	B	AB
1	0.082	0.106	0.008	0.004
2	0.135	0.141	0.018	0.006
3	0.215	0.200	0.065	0.020

Suppose that an individual is randomly selected from the population, and define the events:
A = {Blood Type A selected} B = {Blood Type B selected} C = {Ethnic Group 3 selected}

a) Calculate P(A), P(C), and $P(A \cap B)$
b) Calculate both P(A | C) and P(C | A), and explain in context what each of these probabilities represent.
c) If the selected individual does not have type B blood, what is the probability that he or she is from ethnic group 1?

11) a) If $\boldsymbol{S = \int \frac{sinx}{asinx+bcosx} dx}$ and $\boldsymbol{C = \int \frac{cosx}{asinx+bcosx} dx,}$ find $\boldsymbol{aS + bC}$ and $\boldsymbol{aC - bS}$.

Hence, or otherwise, prove that $\boldsymbol{\int_0^{\frac{\pi}{2}} \frac{sinx}{3sinx+4cosx} dx = \frac{3\pi}{50} + \frac{4}{25} ln\frac{4}{3}}$

b) Sketch the curve $\boldsymbol{y = 4 - \frac{1}{x^2}}$, indicating the asymptotes and the coordinates of the points of intersection of the curve and the X-axis. Find the area in the first quadrant bounded by the curve, the line $x = 2$ and the x-axis.

c) Solve $y'' + 4y' + 4y = 4x - 16$ with $y(0) = -2$ and $y'(0) = -3$

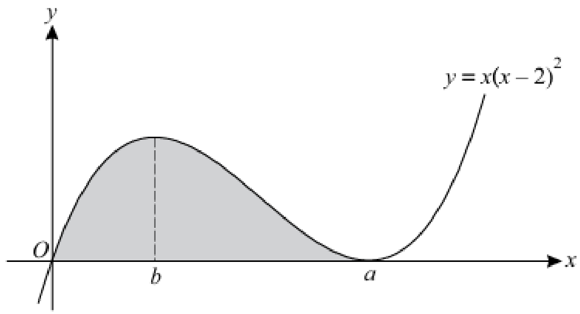
12) Consider the sphere (S) plane (α) and the line (L)

$\boldsymbol{S \equiv x^2 + y^2 + z^2 + 2x - 10y = 23}$ **;**

$\boldsymbol{\alpha \equiv 4x - y - 8z + 1 = 0}$ **and** $\boldsymbol{L \equiv \frac{1}{4}(x+3) = \frac{1}{3}(y+4) = -\frac{1}{5}(z-8)}$ Find

a) The unit normal vector to the plan α
b) The centre and the radius of the sphere
c) The point(s) of contact (if any) of the sphere and the line
d) The acute angle between the plan α and the line L
e) The equation of the circle intersection of the sphere and the plane.

f) The diagram shows the curve with equation $y = x(x - 2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x-coordinate of the maximum point is b, where a and b are constants.



(a) State the value of a. (b) Calculate the value of b.

(c) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m. Calculate the value of m.

(d) calculate the area of the shaded region

13) The circle $x^2 + y^2 + 4x - 2y - 20 = 0$ has centre C and passes through points A and B.

a) State the coordinates of C.

It is given that the midpoint, D, of AB has coordinates $\left(1\frac{1}{2}\ ,1\frac{1}{2}\right)$.

b) Find the equation of AB, giving your answer in the form $y = mx + c$.

c) Find, by calculation, the x-coordinates of A and B

14) i) Consider the function $f : x \mapsto x^2 + ax + b$, where a and b are constants.

a) It is given that $a = 6$ and $b = -8$. Find the range of f.

b) It is given instead that $a = 5$ and that the roots of the equation $f(x) = 0$ are k and $-2k$, where k is a constant. Find the values of b and k.

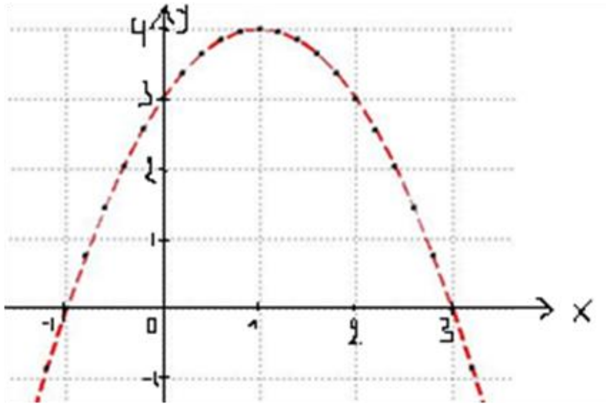
c) Show that if the equation $f(x + a) = a$ has no real roots then $a^2 < 4(b - a)$.

ii) Given that $\boldsymbol{I_n} = \int_0^{\frac{\pi}{2}} \boldsymbol{e^{-nx} sinx\ dx}$ and $\boldsymbol{J_n} = \int_0^{\frac{\pi}{2}} \boldsymbol{e^{-nx} cosx\ dx}$ By applying successive integration by parts on integrals $\boldsymbol{I_n}$and $\boldsymbol{J_n}$,establish two relations between $I_n$and J_n Hence, deduce the value $\boldsymbol{I_1}$and $\boldsymbol{J_1}$

iii) Find the value (s) of x for which $x^3 = 1$

iv) In Probability, the exponential probability distribution function is defined by $\frac{dP}{dt} = \lambda e^{-\lambda t}$ where λ is a positive constant. a) Find an expression for $P(t)$ b) show that $\int_0^{\infty} \lambda e^{-\lambda t}\, dt = 1$

15) Study the graph below and answer the following questions



i) Determine the coordinates of

a) The vertex (pink)

b) Y_intercept

c) x_intercepts

d) write the equation of the axis of symme

e) find the maximum values and the maximum point

ii) Solve

a) f(x) = 0 c)y = 3

b) f(x) > 3

Section B: Objective type questions

16) An accelerometer measures $x(t) = 0.08cos\ (12t)$ m. Find the maximum speed.

A: 0.08 m/s ***B***: 0.96 m/s ***C***: 1.0 m/s ***D***: 12.0 m/s

17) State whether each of the following is TRUE or FALSE

a) The general solution of $sin\,x = sin\,\alpha$ is $x = \alpha + 2k\pi \quad or\ x = \pi - \alpha + 2k\pi$

b) The identity $1 + tan^2\,x = sec^2\,x$ holds for all real x where defined.

c) If light travels from air to glass, it bends away from the normal.

d) Snell's law is given by $n_1 \sin r = n_2 \sin i$

18) State whether each statement is **True (T)** or **False (F)**

a) The exponential population growth is $P(t) = P_0e^{kt}$, where $k > 0$.

b) In population decay, the growth constant k is negative.

c) The formula $P(t) = P_0e^{kt}$ applies only to population growth.

d) The compound interest formula is $A = P\left(1 + \frac{r}{n}\right)^{rt}$.

19) If the sum of the first n terms of a sequence is $S_n = 3n^2 + 5n$ then the nth term is: A. 6n + 5 B. 6n + 2 C. 3n + 5 D. 6n − 3

20) The population growth is $P(t) = P_0e^{kt}$. State whether each statement is **True** or **False**

a) If $k = 0$, the population remains constant for all t.

b) A population doubling time depends only on the initial population.

d) Decay models approach zero but never reach zero

21) Solve the equation $sin2x = sin\, x,\quad 0 \le x < 2\pi.$

$A. x = 0, \pi$ $\quad B.\ x = 0, \frac{\pi}{3}, \pi, \frac{5\pi}{3}$ $\quad$ C. $x = \frac{\pi}{2}, \frac{3\pi}{2}$ $\quad$ D. $x = \frac{\pi}{6}, \frac{5\pi}{6}$

22) Which of the following represents the correct domain of the function $f(x) = \sec x$ $\ A.\ \ \mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$ $\quad B. \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$ $\quad$ c) $[-1,1]$ $\quad D. \mathbb{R}$

23) Match each item in Column A with the correct letter in Column B.

Column A	Column B
1. Interpolation	A. Estimates values outside known data range
2. Bisection method	B. Estimates values within known data range
3. Extrapolation	C. Uses two points to estimate a value
4. Linear interpolation	D. Uses tangent line for root approximation
5. Newton-Raphson method	E. Reduces interval by half each iteration

24) A student claims that the inequality $sinx > cosx$ holds for all $x \in (0, \pi)$. Which evaluation is correct?

A. True, because sine is always positive $\quad$ B. True only in the first quadrant

C. False, because it fails near $x = 0$ $\quad$ D. True for all real values

25) Determine the value of the expression $sin(\arccos\frac{3}{5})$ $\quad A. \frac{3}{5}$ $\quad B. \frac{4}{5}$ $\quad C. \frac{3}{4}$ $\quad D. \frac{5}{4}$

26) What is the domain of the function $f(x) = 3\sin^{-1}(3x - 1)$

$A. \left[0, \frac{2}{3}\right]$ $\quad B. [-1,1]$ $\quad C. \mathbb{R}$ $\quad D. \left[-\frac{2}{3}, 0\right]$ $\quad E. [0,2]$

27) The value of k for which vector $\vec{w} = (1, -2, k)$ of $\mathbb{R}^3$ is a linear combination of vectors $\vec{u}_1 = (3,0,-2)$ and $\vec{u}_2 = (2, -1, -5)$ is

$A.\quad 8 \qquad B.\quad 3 \qquad C.\quad \frac{8}{3} \qquad D.\quad -8 \qquad E.\ neither$

28) If $lim_{h\to 0}\ \left[\frac{arcsin\,(a+h) - arcsin(a)}{h}\right] = \sqrt{2},$

which of the following could be the value of a ?

$A.\ \frac{1}{2} \qquad B.\ \frac{\sqrt{3}}{2} \qquad C.\ \sqrt{3} \qquad D.\ \frac{\sqrt{2}}{2} \qquad E.2$

29) Work out and circle the letter related to the best answer: Consider three vectors in Euclidian space $\mathbb{R}^3$: $u = -3i + 4j + 12k,\ \mathrm{v} = 4i - 3j + 2k\ and\ \ w = mi + 3j - nk$

i) The values of m and n if $u = \mathrm{v} \times w$ are equal to

$A.\begin{cases} m = 0 \\ n = 1 \end{cases} \qquad B.\begin{cases} m = 1 \\ n = 0 \end{cases} \qquad C.\begin{cases} m = 4 \\ n = 12 \end{cases} \qquad D.\begin{cases} m = 12 \\ n = 4 \end{cases}$

ii) The area of the triangle (O, v, w) is

A. 4.5 SU B. 6.5 SU C. 9 SU D. 3 SU

30) i) When the correlation coefficient is zero, the regression lines

A. coincide B. are parallel to axes

C. are perpendicular D. become the same line E. cannot be drawn

ii) Consider the regression line of Y on X: $\boldsymbol{Y = 5 + 2X}$, with standard deviations $s_X = 4, s_Y = 8$.

(a) The slope $b_1$is A. 1 B. 2 C. 4 D. 8 E. 0

(b) The coefficient of correlation r is: A. 0.25 B. 0.5 C. 0.75 D. 1

iii) If regression coefficients are $b_{yx} = 0.5$ and $b_{xy} = 0.8$, then A. $r = 0.65$

B. $r > 1$ C. $r = 0$ D. $r = \sqrt{(0.5 \times 0.8)}$ E. $r = \frac{1}{2}(0.5 + 0.8)$

31) A Ferris wheel of radius 20 m rotates once every 5 minutes. The height h(t) of a seat above ground can be modeled by a sinusoidal function. What is the **maximum height**?

A. 20 m B. 40 m C. 25 m D. 45 m

32) Match **Column A** with the correct principal value range in **Column B**

Column A	Column B
1. $\sin^{-1} x$	A. $[0, \pi]$
2. $\cos^{-1} x$	B. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
3. $\tan^{-1} x$	C. $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$
4. $\sec^{-1} x$	D. $[0, \pi],\ x \neq \pi$
5. $\cot^{-1} x$	E. $(0, \pi)$

33) A function f is increasing if $f'(x) > 0$, and decreasing if $f'(x) < 0$

i. Determine the intervals where $f(x) = x^3 - 3x^2 + 2$ is decreasing.

$a)(-\infty, 0) \cup (2, \infty)$ $\quad b)\ (-\infty, 1) \cup (2, \infty)$

$c)(-\infty, 2) \cup (3, \infty)$ $\quad d)(-\infty, 0) \cup (3, \infty)$

ii. A tide model is $H(t) = 2 + 1.5\cos(\frac{\pi t}{6})$ meters, t in hours. Time for high tide is:

$\boldsymbol{a})\ 3\ hours$ $\quad b)\ 4\ hours$ $\quad c)\ 6\ hours$ $\quad d)\ 12\ hours$

34) The bisection method is applied to solve $f(x) = x^2 - 2$. If the interval is [1, 2], what is the first approximated solution? $a)1.25$ $\quad b)1.5$ $\quad c)1.75$ $\quad d)2$

35) Using Newton-Raphson, solve one iteration for $f(x) = x^2 - 2$ starting at $x_0 = 1.5$.

$a)\ 1.41$ $\quad b)\ 1.45$ $\quad c)1.42$ $\quad d)1.47$

36) A radioactive substance decays with half-life of 10 years. If the initial mass is 200 g, how much remains after 30 years? $a)25\ g$ $\quad b)\ 50\ g$ $\quad c)\ 100\ g$ $\quad c)200\ g$

37) Earthquake A has magnitude 6, Earthquake B has magnitude 8. How many times more intense is B compared to A? $a)10$ $\quad b)100$ $\quad c)1000$ $\quad d)10{,}000$

38) A student sees a bird on top of a 12m high light pole. The student is standing 20m from the base of the pole. At what angle must the student incline her camera to take a picture of the bird? $a)\ 37^\circ$ $\quad b)\ 31^\circ$ $\quad c)\ 59^\circ$ $\quad d)\ 87^\circ$ $\quad e)\ none\ of\ these$

39) Find x if $ln2$, $\ln(2^x - 1)$ and $\ln(2^x + 3)$ are consecutive terms of an arithmetic progression
$a)\ln 5$ $\quad b)2^x$
$c)\log_2 5$ $\quad d)\ln(2^x + 1)$ $\quad e)None\ of\ these$

40) A box has 5 red and 3 blue balls. One is chosen at random. Probability it is blue?

$a)\ \frac{3}{5}$ $b)\frac{5}{8}$ $c)\frac{3}{8}$ $d)\frac{1}{3}$

41) If $tan(\theta) = \frac{3}{4}$, what is sin(θ)? $a)\ \frac{5}{3}$ $b)\ \frac{4}{5}$ $c)\ \frac{3}{4}$ $d)\frac{3}{5}$

42) If x, y and z are the angles between 0 and $\frac{\pi}{2}$ such that $\tan x = \frac{1}{2}, \tan y = \frac{1}{5}$ and $\tan z = \frac{1}{8}$. Find $x + y + z$

$a)\frac{\pi}{6}$ $b)\frac{\pi}{3}$ $c)\frac{\pi}{4}$ $d)\frac{\pi}{2}$ $d)0$ $e)None\ of\ these$

43) A fair die is rolled. Probability of getting a number ≥ 4:

$a)\frac{2}{3}$ $b)\frac{1}{3}$ $c\ )\frac{1}{2}$ $d)\ \frac{3}{4}$

44) A box has 5 red and 3 blue balls. One is chosen at random. Probability it is blue?

$a)\ \frac{3}{5}$ $b)\frac{5}{8}$ $c)\frac{3}{8}$ $d)\frac{1}{3}$

45) A radioactive isotope has half-life 12 hours. If initial mass is 200 g, how much remains after 36 hours?

A. 25 g B. 50 g C. 100 g D. 200 g

46) $\sqrt{-4} \times \sqrt{-9} =$

$a)\ 6$ $b) - 6$ $c)\ - 6i$ $d)6i$

47) A mass–spring system satisfies $x'' + 4x = 0$ with $x(0) = 0.1$m and $x'(0) = 0$. Find x(t).

$\boldsymbol{A}: 0.1cos(2t)$ $\boldsymbol{B}: 0.1sin(2t)$ $\boldsymbol{C}: 0.05cos(2t)$ $\boldsymbol{D}: 0.1cos(t)$

48) Using differentials, approximate $\sqrt{(100.5)}$

$a)\ 10.024$ $b)10.25$ $c)9.95$ $d)10.025$

49) Determine the radius of convergence of $\sum_{n=0}^{\infty} \frac{(-1)^n(x-2)^n}{3^n}$

$a)\ 2$ $b)\frac{2}{3}$ $c)\ 3$ $d)\ \infty$ $e)\ \frac{13}{3}$ $e)\ all\ are\ false$

50) Determine the sum of the series$\sum_{n=2}^{\infty} \frac{6}{n(n+3)}$ or conclude that it diverges. $a)\ 0$ $b)\frac{13}{2}$ $c)\ \frac{5}{3}$ $d)\ \frac{13}{6}$ $e)\ \frac{13}{3}$ $e)\ \frac{10}{2}$ $g)\ Diverges$

51) Determine whether the sequence defined by $a_n = ln(2n^3 + 2) - ln(5n^3 + 2n^2 + 4)$ converges or diverges. If it converges, find the limit

$a)\ 0$ $b) \ln\frac{2}{5}$ $c)\ - \ln\frac{2}{5}$ $d)\ \frac{2}{5}$ $e)\ 2$ $f)\ - 5$ $d)\ Diverges$

…… HAPPY EASTER……